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AFOSR-TR- 89-0606

RADON TRANSFORM ANALYSIS OF A PROBABILISTIC METHOD FOR IMAGE GENERATION

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SUMMARY

The research performed for this grant over the past year involved affine iterated function system (IFS) encoding and IFS mixing for digital images. This relates to a technique of Michael Barnsley's for generating fractal and other images by randomly iterating affine transformations of the plane into itself. By this technique an image is both generated and represented as the long-term probability distribution for a 2–D or 3–D Markov chain. The encoding involves finding an affine "collage" of the image, whereby it is identified as a convex combination of affinely scaled versions of itself. This permits some remarkable data compression. The mixing involves a merging of IFS's so as to produce images with combined textures. It ties in with the encoding in that a broader class of images can then be efficiently encoded, and there are more degrees of freedom in the encoding search. The mathematical methods used involve stochastic optimization, computational geometry, the Radon transform, dynamical systems and ergodic theory for Markov chains. In addition the proposer studied the dynamics of discrete IFS, whereby orbits are truncated so as to always land in centers of pixels. Even in the strictly contractive case, the discrete IFS is not strictly contractive, and uniqueness of stationary distributions is lost. Nevertheless it was shown that as the pixel size goes to zero, any sequence of stationary distributions converges weakly to the stationary measure for the true IFS. Finally, the proposer animated IFS image

generation so as to produce video segments representing flows of images. This is to be used for animation encoding, whereby a dynamical sequence of images is encoded as a time-dependent IFS.

STATEMENT OF WORK

BASIC IFS MODEL: The basic IFS image generation algorithm is illustrated in Fig. 1. The leaf is generated as follows. Pick any point $\mathbf{X}_0 \epsilon \Re_2$. There are four affine transformations $T : \mathbf{x} \mapsto A\mathbf{x} + \mathbf{b}$ listed on the top of this Fig., and four probabilities p_i underneath them. Choose one of these transformations at random, according to the probabilities p_i—say T_k is chosen, and apply it to $\mathbf{X}_0$, thereby obtaining $\mathbf{X}_1 = T_k \mathbf{X}_0$. Then choose a transformation again at random, independent of the previous choice, and apply it to $\mathbf{X}_1$, thereby obtaining $\mathbf{X}_2$. Continue in this fashion, and plot the orbit $\{\mathbf{X}_n\}$. The result is the leaf shown. By tabulating the frequencies with which the points $\mathbf{X}_n$ fall into the various pixels of the graphics window, one can actually plot the empirical distribution $\frac{1}{n+1} \sum_{k=0}^{n} \delta_{\mathbf{X}_k}$, using a grey scale to convert statistical frequency to grey level. The darker portions of the leaf correspond to high probability density.

The following is a brief description of the proposer's mathematical model for IFS theory. Let G be the semi-group of affine transformations $g : \mathbf{x} \mapsto a\mathbf{x} + \mathbf{b}$ from $\Re^m \to \Re^m$. Let $\mu \epsilon P(G)$, the collection of Borel probabilities on G. Given $\nu \epsilon P(\Re^m)$ define the *convolution* $\mu * \nu$ as

$$\mu * \nu(B) = \int \nu(g^{-1}B)\mu(dg)$$

for Borel subsets $B \subseteq \Re^m$. Equivalently if $g \epsilon G$ is distributed like μ, if $\mathbf{X} \epsilon \Re^m$ is distributed like ν and if g and $\mathbf{X}$ are independent, then $\mu * \nu$ is the distribution of $g\mathbf{X}$. Say that $\nu \epsilon P(\Re^m)$ is *μ-stationary* when $\mu * \nu = \nu$. It can be shown that if ν is μ-stationary then

$$C = \overline{\bigcup_{g \epsilon H} gC}$$

where C and H are the supports of ν and μ, respectively. This is the Collage Property. Examples like Fig. 1 correspond to the case where μ is an atomic measure

$$\mu(\{T_i\}) = p_i, \qquad 1 \leq i \leq N. \qquad (*)$$

In this case the stationarity condition becomes

$$\nu(B) = \sum_{i=1}^{n} p_i \nu(T_i^{-1}B)$$

and the Collage Property is

$$C = \overline{\bigcup_{i=1}^{N} T_i C}.$$

This is illustrated in Fig. 2. Observe how C, the grey leaf, is covered by four black leaves—each of which is an affine copy of C.

Let $\{g_n\}$ be an i.i.d. sample from μ. The *Markov chain associated with* μ is defined by

$$\mathbf{X}_{n+1} = g_{n+1}\mathbf{X}_n, \qquad n \geq 0.$$

When μ is the atomic measure given by $(*)$ above, then this chain is precisely the process described in the beginning of this Section. A distribution $\nu \epsilon P(\Re^m)$ is stationary for this chain if and only if it is μ-stationary. The proposer showed in [2], [4] that if μ obeys a suitable contractivity condition, then there exists a unique μ-stationary $\nu \epsilon P(\Re^m)$ and the following Law of Large Numbers holds.

(LLN) *With probability one the empirical distributions* $\frac{1}{n+1}\sum_{k=0}^{n}\delta_{\mathbf{X}_k}$ *converge weakly to* ν.

This LLN ensures that the plot of the orbit of any *single* trajectory $\{\mathbf{X}_n\}$ will produce the desired image.

IFS ENCODING (work with J.-P. Vidal): There are two types of IFS image encoding techniques under development today: interactive computer-aided encoding, and automated encoding. In both instances the objective is to encode a given target digital image so that it can be re-generated as the attractor of an affine IFS. For the interactive encoding one sits down at a terminal and defines various affine transformations geometrically, with the aid of a mouse, by identifying images of triangles or rectangles under the transformations. Probabilities are either assigned to them in proportion to their determinants (area factors), or else user-specified. Then the attractor of the IFS is generated and overlayed upon the target image. Based on this the user can go back and modify his transformations until the attractor fits well with the target. When this occurs the image has been encoded, and in fact the coefficients and probabilities for the affine transformations constitute the code. For images involving roughly ten transformations in the IFS, this interactive encoding typically takes between fifteen and sixty minutes on a micro-Vax, depending on the accuracy desired (assuming that the user is familiar with the Collage Property).

For automated encoding a target digital image is input, and the affine transformations and probabilities for the IFS are all internally calculated. The error between the IFS attractor and the target image is measured by the Hausdorff distance, and the automated encoder searches over affine transformations so as to minimize this error. This type of encoding typically takes from one to ten hours on a micro-Vax.

Algorithms for both types of encoding were developed and coded under this grant, and they are currently being tested and optimized. For the automated encoding the proposer developed two algorithms—one involving extreme points of the convex hull of the image, and one involving sequential constrained optimization. The former was described in the First Annual Technical Report, and the latter goes as follows. At stage i solve for T_i by

$$\text{MAXIMIZE} \qquad [\text{area } (T_iC \bigcap R_{i-1}) - \lambda \text{ area}^2(T_iC - C)]$$

affine contractions T_i

where R_i are the *residual sets* $R_0 = C$,

$$R_i = R_{i-1} - T_iC, \qquad\qquad i \geq 1,$$

and λ is an adjustable adaptive parameter. Furthermore after the transformations $T_1, \ldots, T_N$ have been obtained in this way they can be refined on successive sweeps by modifying the residuals

$$R_{i-1} \leftarrow R_{i-1} \bigcup E_i$$

where

$$E_i = C \bigcap T_iC - \bigcup_{j \neq i} T_jC$$

is the *essential* part of T_iC; namely, those points of C covered exclusively by T_iC. This sequential approach has the nice feature that once T_1 is computed the algorithm automatically stays away from the identity map, since it only tries to cover the residual of C.

IFS MIXING (work with M. Barnsley): During the first year of this grant the proposer [2] developed a mixing algorithm for combining several image textures. This mixing includes both IFS condensation [1, Sec. 3.9] and recurrent IFS [3] as special cases. The analysis of the mixing was only carried out then for the case of two screens (i.e. the mixing of two images). This year the analysis was performed for the general case of N-screen mixing. It involves the asymptotics for products of random affine maps indexed over excursions of a finite state Markov chain from a fixed recurrent state. The mixing process is ergodic if the Lyapunov exponent of such a random product is negative. The proposer is currently working on encoding schemes for mixed processes, and more generally for images which are 2–D or 3–D cross-sections of higher dimensional IFS.

DISCRETE IFS (work with M. Perrugia): Set up window coordinates so that the centers of the pixels are at locations $(\frac{i}{M}, \frac{j}{M})$ for integers i, j. Assume that every point generated by the IFS is sequentially replaced with the center of that pixel in which it lies. The resulting discrete Markov chain is referred to as a *discrete* IFS. If the true IFS is strictly contractive the discrete IFS need not be. Consider for example the 1–D IFS consisting of the single map $T : x \mapsto \frac{2}{3}x$. Suppose $M = 10$ and consider the pixel (0.05, 0.15) which is centered at $x = 0.1$. For this x one has $Tx = 0.0667$, and after rounding one is back to $x = 0.1$. Thus $x = 0, \pm 0.1$ are all absorbing states for the discrete IFS here. The proposer has shown that if the true IFS is strictly contractive then as $M \to \infty$ any sequence of stationary distributions for the discrete IFS converges weakly to the (unique) stationary distribution for the true IFS. Currently he is investigating conditions for ergodicity of the discrete IFS.

IFS ANIMATION

Suppose one has two IFS, say $\{T_i, p_i : 1 \leq i \leq N\}$ and $\{T_i', p_i' : 1 \leq i \leq N\}$. Each of these IFS corresponds to an image and one can set up a continuous flow of images from one to the other by interpolating the IFS parameters. This is by no means the same as pixel interpolation. If ν and ν' are μ and μ'-invariant, respectively, then pixel interpolation would amount to

$$\nu(t) = (1-t)\nu + t\nu', \qquad 0 \leq t \leq 1.$$

This sort of interpolation generally results in blurred intermediate images. The proposer's scheme amounts to letting $\nu(t)$ be the $\mu(t)$-invariant probability, where

$$\mu(t) = (1-t)\mu + t\mu', \qquad 0 \leq t \leq 1.$$

This always produces a clear and distinct sequence of intermediate images.

What makes IFS animation even more special is the ease with which one can rotate, scale, change perspective or vantage point, zoom in and out, or perform any affine transformation on an image. Suppose one wants to apply, say, a 3–D rotation R to the image. This can be incorporated directly into the IFS. Simply replace the original transformations T_i with the composite transformations RT_iR^{-1}. These composite transformations are also affine, and by running the IFS algorithm with them and using the same probabilities as before, the rotation is automatically built in.

IFS animation is a highly parallel algorithm. The images for the various times t in the flow can all be generated in parallel, since there are no dependencies among the images at different times. Furthermore the same sequence of random numbers can be used for all the intermediate images. Figs. 3–5 are some snaps from different animations.

The next step is IFS *encoding for animation.* Here the data compression ratios are enormous, since the encoding of two "endpoint" images suffices to generate the intermediates. Furthermore in certain respects animation encoding is easier than still image encoding. This is because a dynamic sequence of images often exposes more information about the individual still images, such as segmentation information. Velocity tracking of boundaries and key features of an image can be used to decide where to position the temporal IFS linear interpolation points (i.e. to break the animation up into "piecewise linear" video segments), and how to identify the images as IFS mixtures. Animation is potentially the most exciting application of IFS encoding.

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PUBLISHED ARTICLES:

Barnsley, M.F., Berger, M.A., and Soner, H.M., Mixing Markov chains and their images, *Prob. Eng. Inf. Sci.* 2 (1988), 387-414.

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Berger, M.A., Random affine IFS: mixing and encoding, to appear.

Berger, M.A. and Soner, H.M., Random walks generated by affine mappings, *J. Theor. Prob.* 1 (1988), 239-254.

INTERACTIONS: The proposer gave the following presentations:

(1) AFOSR at Bolling Air Force Base on Feb. 24, 1988 (host: Dr. A. Nachman);

(2) NIST in Gaithersburg, MD on March 29, 1989 (host: Dr. F. Sullivan);

(3) NSF in Washington, DC on March 30, 1989 (host: Dr. R. Chin);

(4) Invited talk in Michael Barnsley's minisymposium on chaos at the annual SIAM meeting in Minneapolis, July 10–15, 1988 (host: Dr. M. Barnsley);

(5) Sixth International Conf. on Math. Modelling, held at Washington Univ. in Aug., 1987;

(6) Invited talk for the seminar run by the image processing group (Grenander, McClure, Geman, and Gidas) in the Division of Applied Mathematics at Brown University on Feb. 15, 1989 (host: Dr. B. Gidas);

(7) Invited talk for the seminar run by the Pittsburgh Supercomputing Center in October, 1987 (host: Dr. R. Roskies);

(8) Special 3–day lecture series at Allegheny College, April 10–12, 1989.

PARTICIPATING PROFESSIONALS: The proposer's research on image encoding is being carried out with

(1) Jean-Philippe Vidal (Ph.D. student, Comp. Sci., CMU–funded by AFOSR);

(2) Bill Eddy (Prof. of Statistics, CMU);

(3) Mario Perrugia (Ph.D. student, Statistics, CMU);

(4) Michael Barnsley, John Elton, Jeff Geronimo, and Ron Shonkwiler (Prof.'s of Math., GA Tech.).

The proposer's research on mixing is being carried out with

(1) H. Meté Soner (Prof. of Math., CMU);

(2) Michael Barnsley and John Elton (Prof.'s of Math., GA Tech.).

Computing support has been provided by the Pittsburgh Supercomputing Center (Cray Y-MP/832 and animation equipment), the Statistics Dept. at CMU (micro-Vax and animation/camera equipment) and the Computer Graphics Lab in the School of Mathematics at GA Tech. (Masscomp 5600 series and Encore). An article about the proposer's work on IFS image processing (written by science editor Michael Schneider) will appear in the forthcoming Projects in Scientific Computing for the Pittsburgh Supercomputing Center this June.

$$T_1(x) = \begin{pmatrix} 0.8 & 0 \\ 0 & 0.8 \end{pmatrix} x + \begin{pmatrix} 0.1 \\ 0.04 \end{pmatrix} \qquad T_2(x) = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix} x + \begin{pmatrix} 0.25 \\ 0.4 \end{pmatrix}$$

$$T_3(x) = \begin{pmatrix} 0.355 & -0.355 \\ 0.355 & 0.355 \end{pmatrix} x + \begin{pmatrix} 0.266 \\ 0.078 \end{pmatrix} \qquad T_4(x) = \begin{pmatrix} 0.355 & 0.355 \\ -0.355 & 0.355 \end{pmatrix} x + \begin{pmatrix} 0.378 \\ 0.434 \end{pmatrix}$$

$$p_1 = 0.5 \quad p_2 = 0.168 \quad p_3 = 0.166 \quad p_4 = 0.166$$

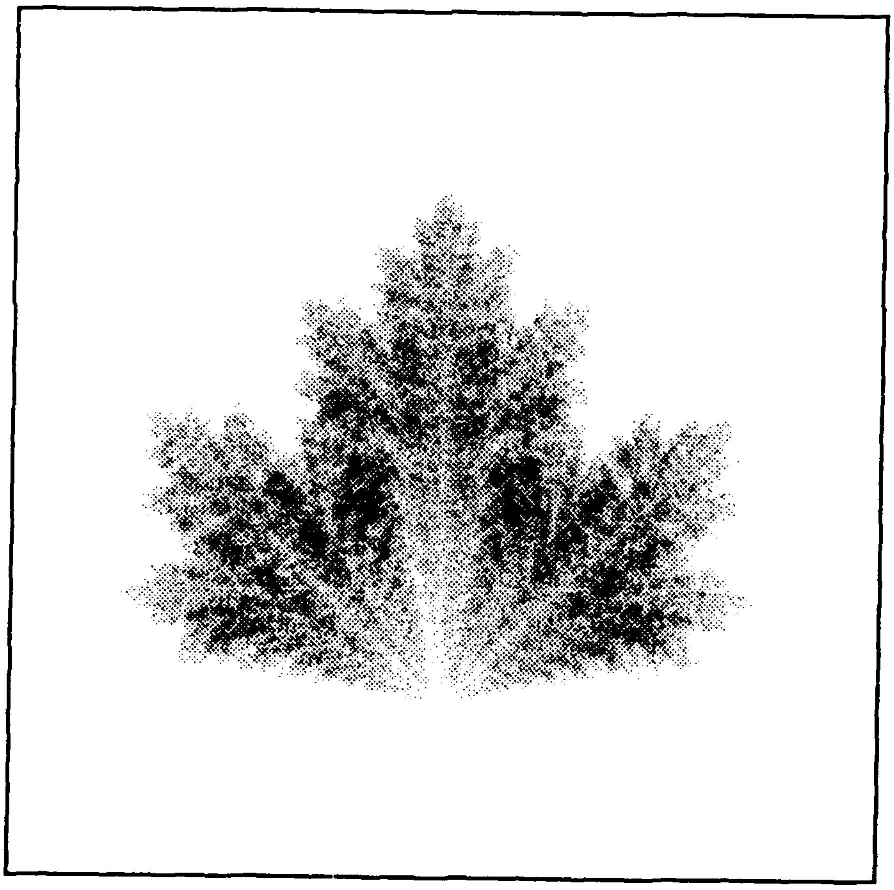

Figure 1: Maple Leaf

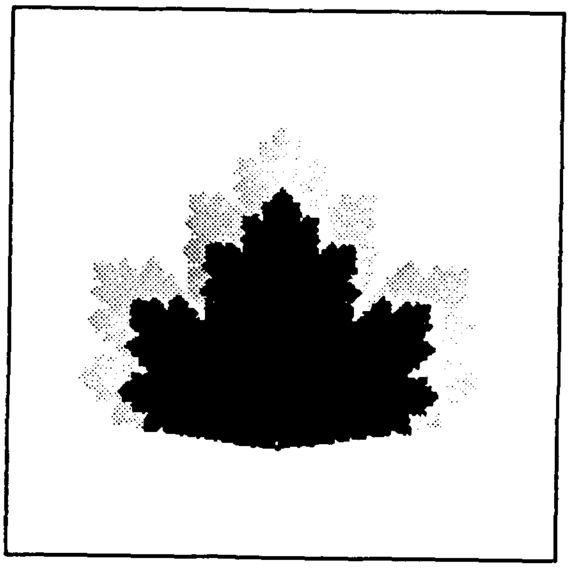

Image under T_1

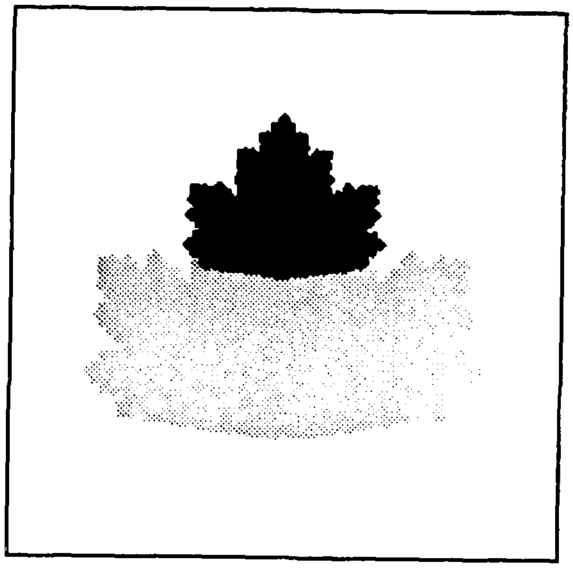

Image under T_2

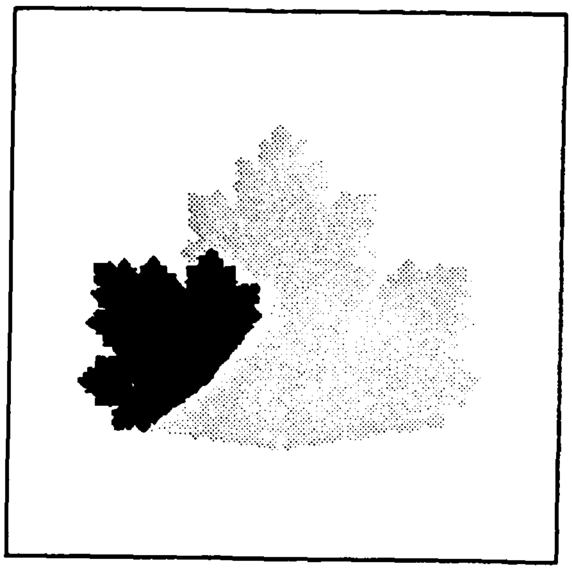

Image under T_3

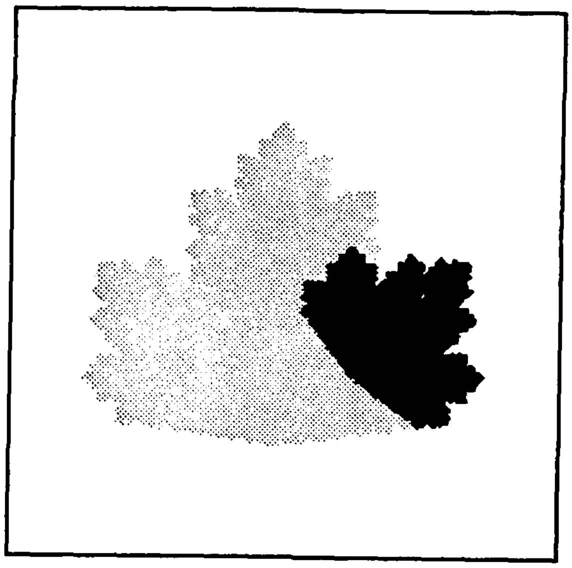

Image under T_4

Figure 2: Collage of the Maple Leaf

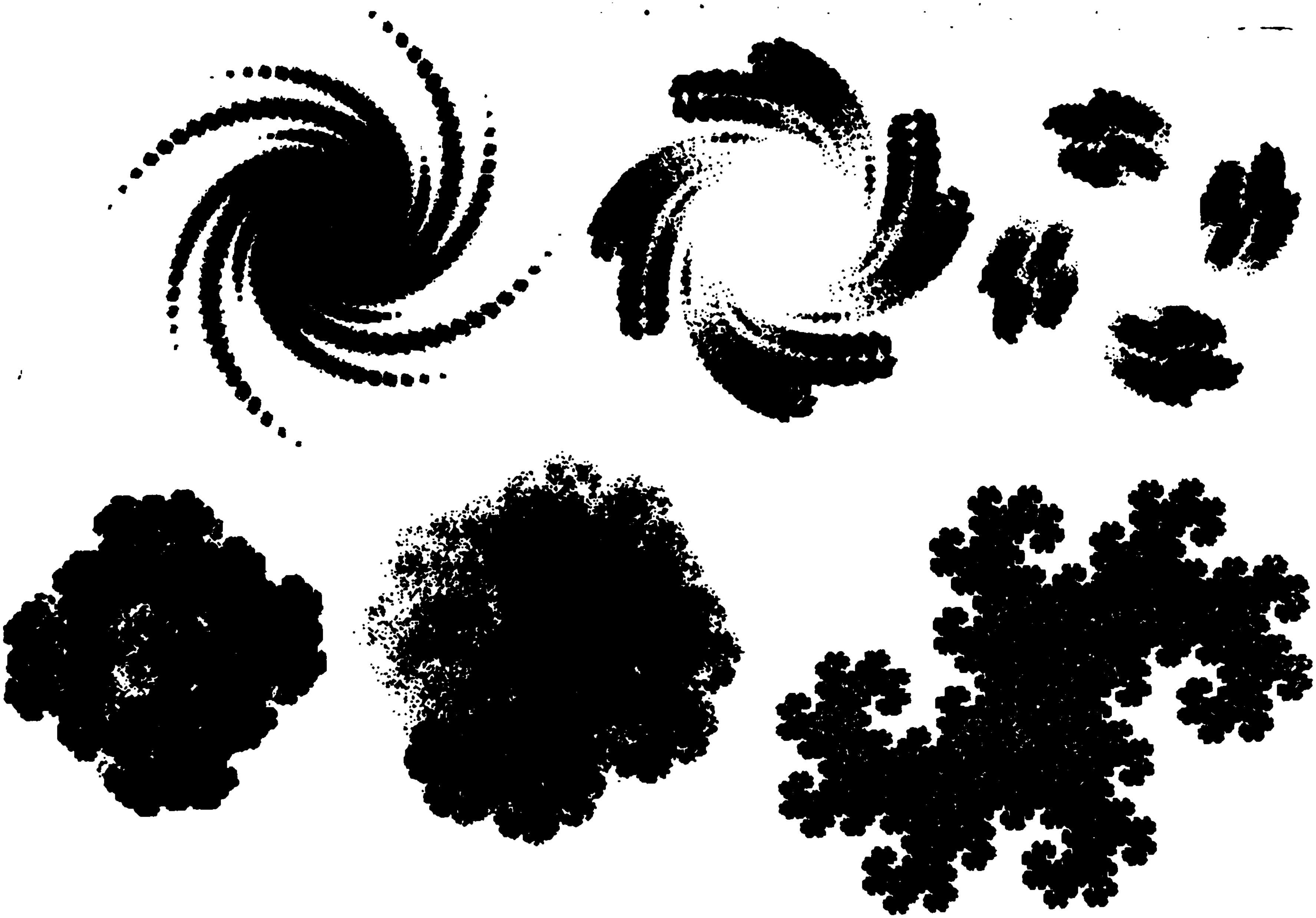

Figure 3: IFS Animation. Scenes from a "walk in parameter space" which flows continuously from the spiral (a) to the twin-dragon (f).

Figure 4: Intermediate images in an image flow from leaf (a) to fern (f). The images depend continuously on the IFS parameters, so that small changes in parameters produce small changes in the image.

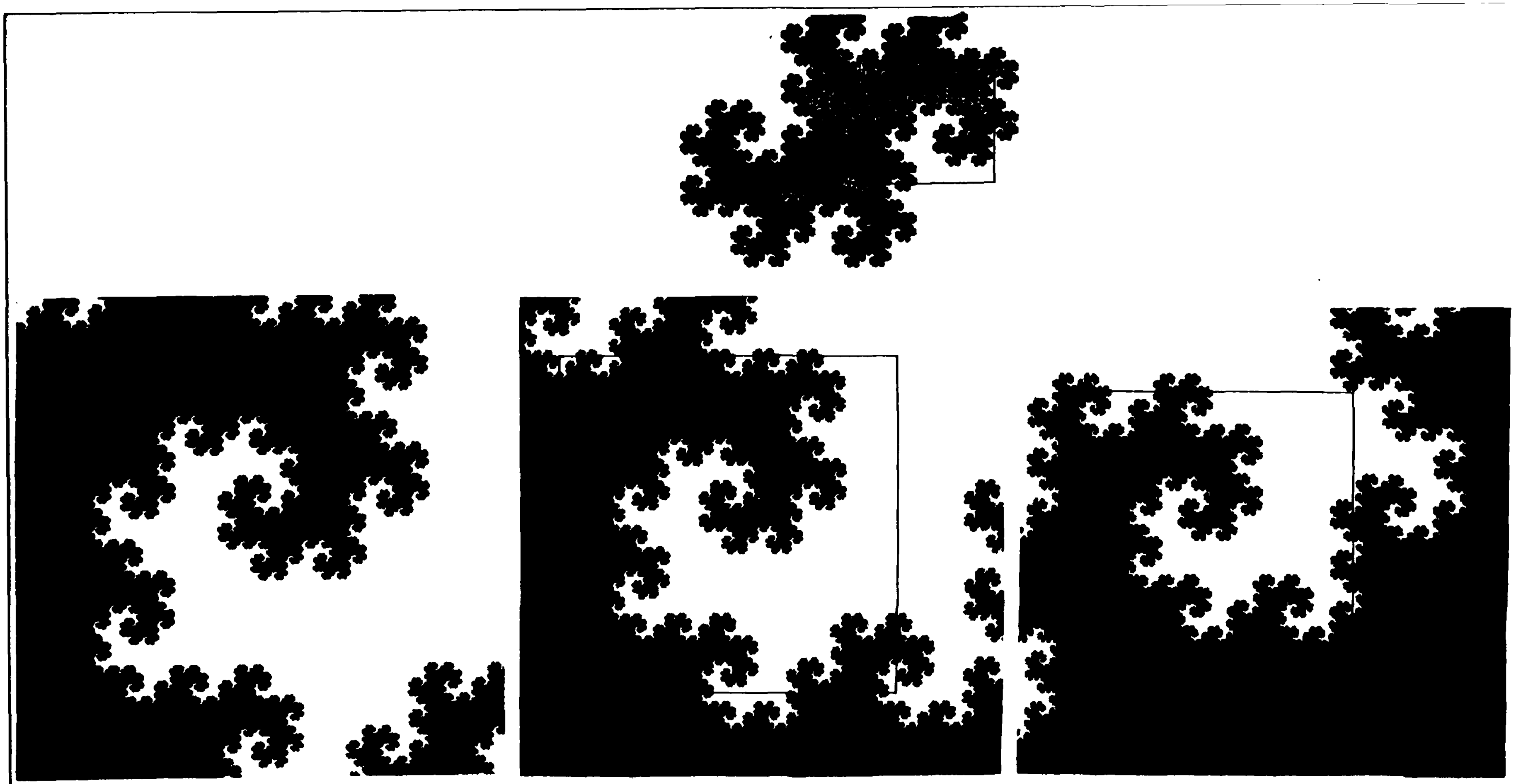

Figure 5: Successive Zoom-In's of the Dragon. The larger the magnification the more points get filtered out of the IFS simulation.